

Interactions in Dark Energy Models

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Abstract

We perform a full dynamical analysis by considering the interactions between dark energy and radiation, and dark energy and dark matter. We find that the interaction helps alleviate the coincidence problem for the quintessence model.

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I. INTRODUCTION

Recent observational data suggest a nearly flat universe with about 0.1% relativistic matter, 30% non-relativistic matter and 70% unknown energy which is called dark energy [1]. The most popular candidate for dynamical dark energy model is the scalar field ϕ [2]. While these models are consistent with current observations, the coincidence problem, which concerns the question why the energy densities of dark energy and matter have the same order of magnitude today even though they evolve very differently, is still the most important puzzle in cosmology. Scalar field with scaling behavior could be used to alleviate the coincidence problem. The scaling solution means that dark energy density scales with the scale factor, so dark energy may track dark matter independent of the initial conditions. However, the attractor of those models is totally dark energy domination $\Omega_\phi = 1$. By considering the interaction between dark energy and matter, it is possible to get the accelerating attractor with constant ratio between the energy densities of dark energy and dark matter, so the coincidence problem could be alleviated.

Although the radiation is negligible today, it played an important role in early universe. The existence of radiation leads to the so-called “cosmic triple coincidence”: why the energy density of radiation today is three orders of magnitude smaller than that of dark matter or dark energy [3]. Borrowing the method of solving the first coincidence problem, we consider the interaction between radiation and dark energy to alleviate the “cosmic triple coincidence” problem. Although the Big Bang Nucleosynthesis (BBN) imposes a very strong constraint on radiation, it is possible that a new kind of radiation called dark radiation may exist after BBN [4]. Therefore, it is possible to add an interaction between radiation and dark energy after BBN.

To address the coincidence problem, we need to find accelerating attractor solution with order one ratio for Ω_m/Ω_ϕ because attractor solution is independent of initial conditions, so we apply the dynamical analysis of phase-space with the introduction of interactions between dark energy, dark matter and radiation. The phase-space analysis is a standard method to study the stability of the fixed points [5–9].

The letter is organized as follows. In section II, we introduce our dark energy model. In section III, the results of phase-space analysis are obtained. Conclusion is presented in section IV.

II. THE INTERACTIONS BETWEEN COSMIC COMPONENTS

Based on the flat Friedmann-Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1}^3 (dx^i)^2, \quad (1)$$

where a is the scale factor, Friedmann equation is

$$H^2 = \frac{1}{3m_{pl}^2}(\rho_\phi + \rho_r + \rho_m), \quad (2)$$

where $H = \dot{a}/a$ is Hubble parameter, $m_{pl}^2 = (8\pi G)^{-1}$, ρ_ϕ , ρ_m , and ρ_r denote the energy densities of dark energy which is assumed to be a scalar field, pressureless matter, and radiation, respectively. Writing $\rho_{tot} = \rho_\phi + \rho_m + \rho_r$, the equation of energy conservation is,

$$\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0. \quad (3)$$

If there is no interaction between dark energy and the other field in the universe, then it is almost impossible to detect dark energy through standard technique without gravitational effect involved. Therefore, we consider the interactions between dark energy and the other matter components in the universe.

For the interaction between dark energy and radiation, we assume that,

$$\dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2, \quad (4)$$

where the interaction term $\Gamma\dot{\phi}^2$ is the source of radiation and Γ is the dissipative coefficient which is related to the microscopic physics of the interaction. This interaction form was first proposed in warm inflation [10–14] based on supersymmetry (SUSY). If $\Gamma > 0$, the energy transfer from dark energy to radiation makes the radiation to decrease slower and dark energy to decrease faster, therefore provides the possibility to alleviate the coincidence problem.

Motivated by the different forms of the dissipative coefficients in Refs. [10–14], a general form of Γ was proposed in Ref. [13],

$$\Gamma = C_\phi \frac{T^{\tilde{m}}}{\phi^{\tilde{m}-1}}, \quad (5)$$

where C_ϕ and \tilde{m} are parameters related to the dissipative microscopic dynamics. Moreover, a more general phenomenological form of Γ was proposed in Ref. [14],

$$\Gamma = C_\phi \frac{T^m}{\phi^n}, \quad (6)$$

where m and n are constants. To be more realistic, the interaction between dark energy and radiation is assumed to be turned on after the BBN period.

For the interaction between dark energy and matter, we consider the interaction form which is proportional to $\rho_m \dot{\phi}$, the energy conservation equation of ρ_m is

$$\dot{\rho}_m + 3H\rho_m = \frac{\alpha\rho_m\dot{\phi}}{m_{pl}}, \quad (7)$$

the interaction form on the right hand side was motivated from Brans-Dicke theory and is widely used to alleviate the coincidence problem [16]. If $\alpha > 0$, the energy transfer from dark energy to dark matter makes dark matter to decrease slower and dark energy to decrease faster. Therefore, the interactions based on Eqs. (4) and (7) provide the possibility of alleviating the “cosmic triple coincidence” problem.

There are evidences that the equation of state (EoS) parameter of dark energy crosses over the value of -1 [17], so we consider both quintessence and phantom cases. The energy density and pressure of dark energy are written as

$$\rho_\phi = \epsilon \frac{\dot{\phi}^2}{2} + V(\phi), \quad (8)$$

$$p_\phi = \epsilon \frac{\dot{\phi}^2}{2} - V(\phi), \quad (9)$$

where the sign $\epsilon = \pm 1$. If $\epsilon = 1$, then the scalar field is the quintessence field; if $\epsilon = -1$, then the scalar field is the phantom field.

Combining the energy conservation Eqs. (3), (4) and (7), the equation of motion of the scalar field becomes

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + \epsilon V_{,\phi} = -\epsilon \frac{\alpha\rho_m}{m_{pl}}, \quad (10)$$

where $V_{,\phi}$ means the partial derivative of the potential V with respect to the scalar field ϕ .

The exponential potential is one of the most often used potential in cosmology [18–20]. In higher-dimensional gravitational theories such as superstring and Kaluza-Klein theories, exponential potentials are often obtained by the compactification of the internal spaces of extra dimensions. Moreover, it is known that exponential potentials can arise from gaugino condensation as a non-perturbative effect, and from supergravity corrections in global supersymmetric theories. The possible role of exponential potentials used to solve the coincidence problem was studied extensively, and the scaling solution was found for the exponential potential. In this letter, we consider the exponential potential for the scalar field,

$$V = V_0 e^{-\lambda\phi/m_{pl}}, \quad (11)$$

where λ and V_0 are the model parameters. If the scaling solution still exists in the interacting model, then the “triple coincidence problem” may be alleviated. In the following, to investigate the coincidence problem, we present a phase-space analysis.

III. THE PHASE-SPACE ANALYSIS

We define the dimensionless parameters as

$$\begin{aligned} x &= \frac{\dot{\phi}}{\sqrt{6}m_{pl}H}, & y &= \frac{\sqrt{V}}{\sqrt{3}m_{pl}H}, & z &= \frac{\sqrt{6}m_{pl}}{\phi}, \\ u &= \frac{\sqrt{\rho_r}}{\sqrt{3}m_{pl}H}, & v &= \frac{\sqrt{\rho_m}}{\sqrt{3}m_{pl}H}. \end{aligned} \quad (12)$$

By using these dimensionless parameters, we get $\Omega_m = v^2$, $\Omega_r = u^2$, $\Omega_\phi = \epsilon x^2 + y^2$ and $w_\phi = (\epsilon x^2 - y^2)/(\epsilon x^2 + y^2)$. The dynamical evolution of the universe becomes

$$x' = -x \frac{\dot{H}}{H^2} - \frac{(3H + \Gamma)\dot{\phi} + \epsilon V_{,\phi} + \epsilon \alpha \rho_m / m_{pl}}{\sqrt{6}m_{pl}H^2}, \quad (13)$$

$$y' = -y \frac{\dot{H}}{H^2} + \frac{\dot{\phi}}{\sqrt{6}m_{pl}H} \frac{V_{,\phi}}{\sqrt{2V}H}, \quad (14)$$

$$z' = -\frac{6m_{pl}^2}{\phi^2} \frac{\dot{\phi}}{\sqrt{6}m_{pl}H}, \quad (15)$$

$$u' = -u \frac{\dot{H}}{H^2} - 2u + \frac{\Gamma \dot{\phi}^2}{2\sqrt{3}\rho_r m_{pl}H^2}, \quad (16)$$

$$v' = -v \frac{\dot{H}}{H^2} - \frac{3}{2}v + \frac{\alpha \sqrt{\rho_m} \dot{\phi}}{2\sqrt{3}(m_{pl}^2 H^2)}. \quad (17)$$

where a prime means the derivative with respect to $\ln a$.

In addition, the evolution of the Hubble parameter could be expressed as

$$-\frac{\dot{H}}{H^2} = 3\epsilon x^2 + 2u^2 + \frac{3}{2}v^2. \quad (18)$$

After substituting the above equation into the dynamical Eqs. (13)-(17) of the dimensionless parameters, the following term becomes the only one that needs to be expressed as the dimensionless parameter,

$$\frac{\Gamma}{H} = \frac{C_\phi}{H} \left(\frac{\sqrt{3}m_{pl}Hu}{\sqrt{C_r}} \right)^{m/2} \left(\frac{z}{\sqrt{6}m_{pl}} \right)^n, \quad (19)$$

where $C_r = \pi^2 g^*/30$ and g^* is the effective number of degrees of freedom for the relativistic species in the universe. If $m = 2$, the dynamical system could be written as an autonomous system, and the evolutions of the dimensionless parameters are

$$x' = -3x + \epsilon\lambda\sqrt{\frac{3}{2}}y^2 + x(\epsilon 3x^2 + 2u^2 + \frac{3}{2}v^2) - Kxz^n u - \epsilon\alpha\sqrt{\frac{3}{2}}v^2, \quad (20)$$

$$y' = -\lambda\sqrt{\frac{3}{2}}xy + y(\epsilon 3x^2 + 2u^2 + \frac{3}{2}v^2), \quad (21)$$

$$z' = -z^2 x, \quad (22)$$

$$u' = u(\epsilon 3x^2 + 2u^2 + \frac{3}{2}v^2) - 2u + Kx^2 z^n, \quad (23)$$

$$v' = v(\epsilon 3x^2 + 2u^2 + \frac{3}{2}v^2) - \frac{3}{2}v + \alpha\sqrt{\frac{3}{2}}xv. \quad (24)$$

where $K = -\sqrt{2C_r}(6m_{pl}^2)^{(n-1)/2}/C_\phi$. The critical points of the autonomous system could be derived by imposing the conditions $x' = y' = z' = u' = v' = 0$. Nevertheless, since Friedman equation is

$$\epsilon x^2 + y^2 + u^2 + v^2 = 1, \quad (25)$$

which acts as a constraint equation, there are only four independent variables, we choose them as x , y , z and u . After obtaining the fixed points, we perturb the four independent dynamical equations of Eqs. (20)-(24) around the fixed points, and then linearize the perturbed equations, the four eigenvalues of the coefficient matrix of the linearized equations determine the stability of the corresponding critical point. When the real parts of the eigenvalues are all positive, the corresponding critical point is an unstable fixed point. When the real parts of the eigenvalues are all negative, the corresponding critical point is a stable attractor point. When some of the real parts of the eigenvalues are negative, and some of the real parts of the eigenvalues are positive, the corresponding critical point is an unstable saddle point [5].

A. The Quintessence Case

In the quintessence case, the EoS parameter could be expressed as

$$\omega_\phi = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} = \frac{x^2 - y^2}{x^2 + y^2}. \quad (26)$$

Based on the dynamical Eqs. (20)-(24), we derive the critical points and list their properties in TABLE I. When radiation is absent, we recover the results given by Boehmer et al. [7].

The critical points are explained as follows.

R_q The radiation dominated phase;

For this fixed point, $\Omega_r = 1$, the eigenvalues are $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 0$, and $\lambda_4 = -1 - Kz^n$. Because $\lambda_1 > 0$ and $\lambda_2 > 0$, it is an unstable fixed point. In particular, if $\lambda_4 < 0$, then R_q is an unstable saddle point. Therefore, the universe does not always stay radiation dominated phase and it will go to matter dominated phase.

S_{qR} The deceleration phase with radiation and scalar field;

For this fixed point, $\Omega_r = 1 - 4/\lambda^2$, $\Omega_\phi = 4/\lambda^2$ and $w_\phi = 1/3$. The scalar field tracks the dynamics of radiation, with a constant ratio between the two energy densities. For this critical point, we choose x, y, z, v as the independent dynamical parameters. The eigenvalues are $\lambda_1 = 0$, $\lambda_2 = 1/2 + 2\alpha/\lambda$, $\lambda_3 = -1/2 + \sqrt{-15/4 + 16/\lambda^2}$, and $\lambda_4 = -1/2 - \sqrt{-15/4 + 16/\lambda^2}$. If $\alpha/\lambda > -1/4$ or $\lambda^2 < 4$, the fixed point is an unstable saddle point. Since the fixed point is unstable and Ω_ϕ/Ω_r is a constant, when $\alpha/\lambda > -1/4$ and $\lambda^2 > 4$, once the universe exits from this phase with constant Ω_r/Ω_ϕ and enters into the accelerating phase, the coincidence problem becomes weakened.

M_q The deceleration phase with matter and scalar field;

For this fixed point, $\Omega_m = 1 - 2\alpha^2/3$, $\Omega_\phi = 2\alpha^2/3$ and $w_\phi = 1$. When the interaction is absent, $\alpha = 0$, it becomes the matter dominated phase and it is an unstable saddle point. If the interaction exists, the ratio Ω_ϕ/Ω_m is a constant, but there is no acceleration since $w_\phi = 1$. The eigenvalues are $\lambda_1 = 0$, $\lambda_2 = \alpha^2 - 1/2$, $\lambda_3 = -3/2 + \alpha^2$, and $\lambda_4 = \lambda\alpha + \alpha^2 + 3/2$. If $\alpha^2 > 1/2$, or $\lambda^2 < 6$, or $\lambda^2 \geq 6$ and $(-\lambda + \sqrt{\lambda^2 - 6})/2 < \alpha < \sqrt{6}/2$, this point is an unstable fixed point. In this phase, the scalar field cannot represent dark energy, but it can provide the correct order of magnitude for the ratio Ω_ϕ/Ω_m . So if the universe enters the dark energy dominated phase after this phase, the coincidence problem becomes weakened, i.e., the interaction helps weaken the coincidence problem.

S_{qRM} The deceleration phase with matter and radiation;

For this fixed point, $\Omega_m = 1/3\alpha^2$, $\Omega_r = 1 - 1/2\alpha^2$, $\Omega_\phi = 1/6\alpha^2$ and $w_\phi = 1$. Since $\alpha^2 > 1/2$, this phase requires strong coupling between dark sectors, $\Omega_m/\Omega_\phi = 2$ and $\Omega_\phi/\Omega_r = 1/(6\alpha^2 - 3)$. The eigenvalues are $\lambda_1 = 0$, $\lambda_2 = 2 + \lambda/2\alpha$, $\lambda_3 = -1/2 +$

$\sqrt{-3/4 + 1/2\alpha^2}$ and $\lambda_4 = -1/2 - \sqrt{-3/4 + 1/2\alpha^2}$. When $\lambda/\alpha > -4$, this point is an unstable fixed point, so the universe may exit from this phase and enter into the accelerating phase. Because of nonzero values of Ω_ϕ/Ω_m and Ω_ϕ/Ω_r , the interaction helps weaken the triple coincidence problem.

K_{q+} The kinetic term dominated phase A;

For this fixed point, $\Omega_\phi = 1$ and $w_\phi = 1$, it is a deceleration phase which is not physically interesting. The eigenvalues are $\lambda_1 = 1$, $\lambda_2 = 0$, $\lambda_3 = 3 - \lambda\sqrt{3/2}$, and $\lambda_4 = 3 + 2\alpha\sqrt{3/2}$, so this point is an unstable fixed point. In particular, when $\alpha > -\sqrt{3/2}$ and $\lambda < \sqrt{6}$, all the eigenvalues are positive.

K_{q-} The kinetic term dominated phase B;

For this fixed point, $\Omega_\phi = 1$ and $w_\phi = 1$, it is also a deceleration phase which is not physically interesting. The eigenvalues are $\lambda_1 = 1$, and $\lambda_2 = 0$, $\lambda_3 = 3 + \lambda\sqrt{3/2}$, and $\lambda_4 = 3 - 2\alpha\sqrt{3/2}$, so this point is an unstable fixed point as well. In particular, when $\alpha < \sqrt{3/2}$ and $\lambda > -\sqrt{6}$, all the eigenvalues are positive.

S_q The dark energy dominated phase;

For this fixed point, $\Omega_\phi = 1$ and $w_\phi = -1 + \lambda^2/18$. The eigenvalues are $\lambda_1 = 0$, $\lambda_2 = -2 + \lambda^2/2$, $\lambda_3 = -3 + \lambda^2/2$ and $\lambda_4 = -3 + \lambda^2 + \lambda\alpha$. The stability condition requires that all the eigenvalues are negative which means $\lambda^2 < 4$ and $\lambda\alpha < (3 - \lambda^2)$. However, this attractor does not solve the coincidence problem.

S_{qM} The scaling solution with nonzero Ω_m and Ω_ϕ ;

For this fixed point, $\Omega_m = (\lambda(\lambda + \alpha) - 3)/(\lambda + \alpha)^2$, $w_\phi = -\alpha(\lambda + \alpha)/(\alpha(\lambda + \alpha) + 3)$, and Ω_m/Ω_ϕ is a constant. The existence condition of the fixed point is $\alpha(\lambda + \alpha) > -3/2$ and $\lambda(\lambda + \alpha) > 3$ (hereafter Condition 1). The eigenvalues are $\lambda_1 = 0$, $\lambda_2 = -1/2 - 3\alpha/[2(\lambda + \alpha)]$, $\lambda_3 = -3/2 - 3\alpha/2(\lambda + \alpha) + a$ and $\lambda_4 = -3/2 - 3\alpha/2(\lambda + \alpha) - a$, where $a = \sqrt{9/16 + 9\alpha^2/16(\alpha + \lambda)^2 + 81\alpha/8(\lambda + \alpha) + 27/2(\lambda + \alpha)^2 - 9\lambda/2(\lambda + \alpha) - 3\lambda\alpha}$. If there is no interaction between the dark sectors, we recover the standard tracking solution $w_\phi = w_m = 0$ [19]. If $\alpha(\alpha + \lambda) \gg 3$, $w_\phi \sim -1$, we get a dark energy dominated solution. In FIG. 1, we plot the regions of the parameters α and λ that make the fixed point to be stable. To illustrate the stability of the fixed point, we plot the phase trajectories with $\lambda = 5$, $\alpha = 0.01$, $n = 4$, $K = 0.1$ in FIG. 2. By choosing

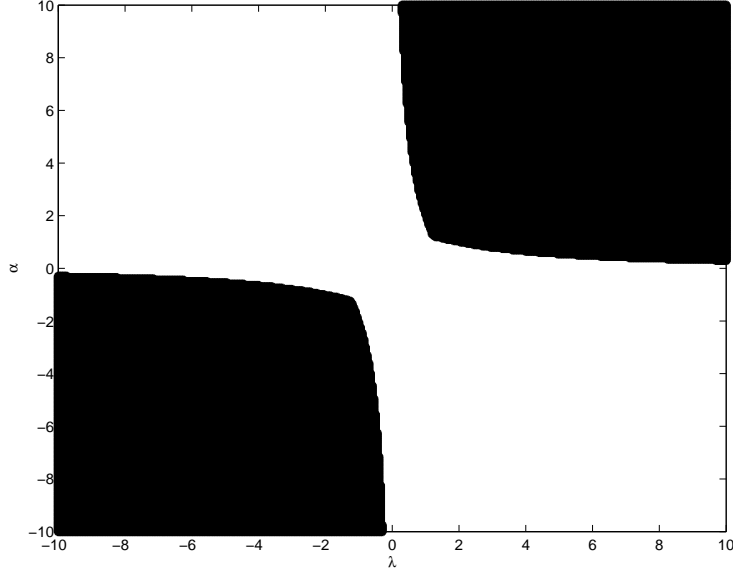


FIG. 1: The parameter space for the point S_{qM} to be stable.

appropriate parameters, we get dark energy dominated phase with $\Omega_m/\Omega_\phi \approx 3/7$, so the coincidence problem can be alleviated.

For the quintessence case, we have several physical interesting unstable fixed points. Immediately after the total radiation dominated phase R_q , the phase S_{qR} provides the possibility of nonzero Ω_ϕ at early time and helps weaken the coincidence problem, then the phase M_q provides the matter domination with small Ω_ϕ which also helps weaken the coincidence problem, it follows by the phase S_{qRM} which helps weaken the triple coincidence problem, finally the universe enter the dark energy dominated phase S_{qM} with the observed ratio between dark energy and dark matter.

B. The Phantom Case

For the phantom case, the EoS parameter of the scalar field is

$$\omega_\phi = \frac{-\dot{\phi}^2/2 - V}{-\dot{\phi}^2/2 + V} = \frac{-x^2 - y^2}{-x^2 + y^2}. \quad (27)$$

The properties of the critical points are summarized in TABLE II. When radiation is absent, we recover the results given by Leon and Saridakis [8].

R_p The radiation dominated phase;

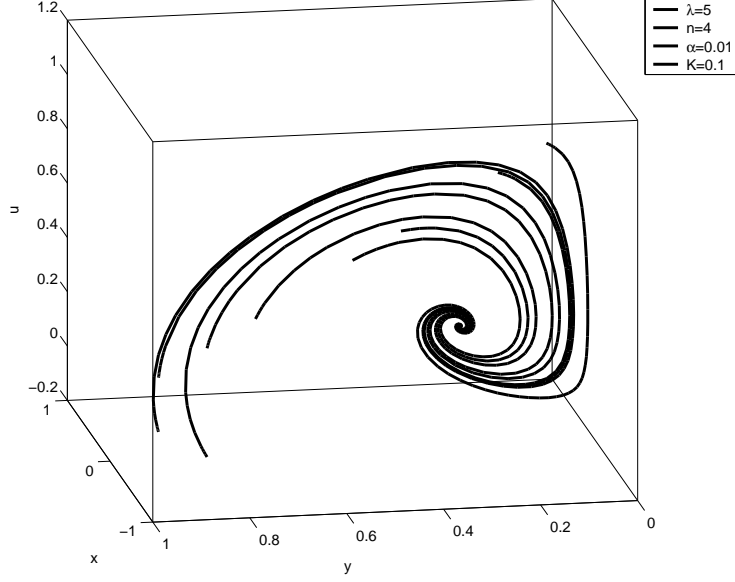


FIG. 2: The phase-space trajectories with $\lambda = 5$, $\alpha = 0.01$, $n = 4$, $K = 0.1$, the stable fixed point is S_{qM} .

	(x, y, z, u, v)	existence	w_ϕ	Ω_ϕ	Acceleration
R_q	$(0, 0, z, 1, 0)$	all λ	0	0	No
M_q	$(-\sqrt{\frac{2}{3}}\alpha, 0, 0, 0, \sqrt{1 - \frac{2\alpha^2}{3}})$	$\alpha^2 < \frac{3}{2}$	1	$2\alpha^2/3$	No
K_{q+}	$(1, 0, 0, 0, 0)$	all λ	1	1	No
K_{q-}	$(-1, 0, 0, 0, 0)$	all λ	1	1	No
S_q	$(\frac{\lambda}{\sqrt{6}}, \sqrt{1 - \frac{\lambda^2}{6}}, 0, 0, 0)$	$\lambda^2 < 6$	$-1 + \frac{\lambda^2}{18}$	1	$\lambda^2 < 6$
S_{qM}	$(\sqrt{\frac{3}{2(\lambda+\alpha)^2}}, \sqrt{\frac{2\alpha(\lambda+\alpha)+3}{2(\lambda+\alpha)^2}}, 0, 0, \sqrt{\frac{\lambda(\lambda+\alpha)-3}{(\lambda+\alpha)^2}})$	Condition 1	$-\frac{\alpha(\lambda+\alpha)}{\alpha(\lambda+\alpha)+3}$	$\frac{3+\alpha(\lambda+\alpha)}{(\lambda+\alpha)^2}$	$\frac{1}{\alpha(\lambda+\alpha)} < \frac{2}{3}$
S_{qRM}	$(-\frac{1}{\sqrt{6}\alpha}, 0, 0, \sqrt{1 - \frac{1}{2\alpha^2}}, \frac{1}{\sqrt{3}\alpha})$	$\alpha^2 > \frac{1}{2}$	1	$\frac{1}{6\alpha^2}$	No
S_{qR}	$((\frac{8}{3\lambda^2})^{1/2}, (\frac{4}{3\lambda^2})^{1/2}, 0, \sqrt{1 - \frac{4}{\lambda^2}}, 0)$	$\lambda^2 > 4$	1/3	$\frac{4}{\lambda^2}$	No

TABLE I: The properties of the critical points when $m = 2$ in the quintessence case.

For this fixed point, $\Omega_r = 1$ and $w_\phi = 1$. The eigenvalues are $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 0, \lambda_4 = -1 - Kz^n$, so this point is an unstable fixed point.

M_p The matter dominated phase;

For this fixed point, $\Omega_\phi = -2\alpha^2/3$ and $w_\phi = 1$. The eigenvalues are $\lambda_1 = 0, \lambda_2 = -\alpha^2 - 1/2, \lambda_3 = -3/2 - 2\alpha^2 - 3\alpha^2$, and $\lambda_4 = \lambda\alpha - \alpha^2 + 3/2$. This point exists only

	(x, y, z, u, v)	existence	w_ϕ	Ω_ϕ	Acceleration
R_p	$(0, 0, z, 1, 0)$	All λ	1	0	No
M_p	$(\sqrt{\frac{2}{3}}\alpha, 0, 0, 0, \sqrt{1 + \frac{2\alpha^2}{3}})$	$\alpha = 0$	1	$-\frac{2}{3}\alpha^2$	No
S_p	$(-\frac{\lambda}{\sqrt{6}}, \sqrt{1 + \frac{\lambda^2}{6}}, 0, 0, 0)$	All λ	$-\frac{\lambda^2}{18} - 1$	1	All λ
S_{pM}	$(\frac{\sqrt{6}}{2(\lambda+\alpha)}, \sqrt{\frac{-3+2\alpha(\lambda+\alpha)}{2(\lambda+\alpha)^2}}, 0, 0, \sqrt{1 + \frac{3-\alpha(\lambda+\alpha)}{(\lambda+\alpha)^2}})$	Condition 2	$\frac{-\alpha(\lambda+\alpha)}{-3+\alpha(\lambda+\alpha)}$	$\frac{-3+\alpha(\lambda+\alpha)}{(\lambda+\alpha)^2}$	$\frac{-1}{\alpha(\alpha+\lambda)} < \frac{2}{3}$

TABLE II: The properties of the critical points when $m = 2$ in the phantom case.

when $\alpha = 0$, and it is an unstable fixed point.

S_p The phantom dominated phase;

For this fixed point, $\Omega_\phi = 1$ and $w_\phi = -\lambda^2/18 - 1$. The eigenvalues are $\lambda_1 = 0$, $\lambda_2 = -2 - \lambda^2/2$, $\lambda_3 = -3 - \lambda^2 - \alpha\lambda$ and $\lambda_4 = -\lambda^2/2 - 3$. If $\alpha^2 < 12$, or $\alpha^2 > 12$ and $\lambda > (-\alpha + \sqrt{\alpha^2 - 12})/2$ or $\lambda < (-\alpha - \sqrt{\alpha^2 - 12})/2$, $\lambda_3 < 0$, the point is a stable fixed point. Since $\Omega_\phi = 1$ does not solve the coincidence problem, we do not discuss the point in detail.

S_{pM} The scaling phase with acceleration;

For this fixed point, $\Omega_\phi = (-3 + \alpha(\lambda + \alpha))/(\lambda + \alpha^2)$, $w_\phi = -\alpha(\lambda + \alpha)/(-3 + \alpha(\lambda + \alpha))$ and $\Omega_m = 1 - \Omega_\phi$. The existence condition of the fixed point is $\alpha(\lambda + \alpha) > 3$ and $\lambda(\lambda + \alpha) > -3$ (hereafter Condition 2). The eigenvalues are $\lambda_1 = 0$, $\lambda_2 = -1/2 - 3\alpha/2(\lambda + \alpha)$, $\lambda_3 = -3/2 - 3\alpha/2(\lambda + \alpha) + b$ and $\lambda_4 = -3/2 - 3\alpha/2(\lambda + \alpha) - b$, where $b = \sqrt{9/16 + 3\alpha\lambda - 27/2(\lambda + \alpha)^2 - 9\lambda/2(\lambda + \alpha) + 9\alpha^2/16(\alpha + \lambda)^2 + 81\alpha/8(\alpha + \lambda)}$. The point is an unstable fixed point.

Therefore, the interacting phantom model cannot alleviate the coincidence problem.

IV. CONCLUSION

We consider radiation, matter and dark energy and the interactions between them, we then did a full dynamical analysis for both quintessence and phantom cases. For the quintessence case, we find that the unstable fixed points S_{qR} , M_q and S_{qRM} help weaken the coincidence problem. In particular, S_{qR} provides small Ω_ϕ after the totally radiation dominated ($\Omega_r = 1$) era, M_q provides the matter dominated era with nonzero Ω_ϕ , and S_{qRM}

helps weaken the triple coincidence problem and prepares the universe for the dark energy domination era S_{qM} . The interaction between dark matter and dark energy helps alleviate the coincidence problem for the quintessence case. However, the interaction cannot alleviate the coincidence problem for the phantom case. The interaction between radiation and dark energy does not affect the dynamical behavior of the universe.

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